# Appointment-Generated Arrival Processes

(as in outpatient scheduling in healthcare)

IEOR 4615, Service Engineering, Professor Whitt

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**[two 2015 papers]**

We are also interested in the service and how it is provided, and the associated operational efficiency (queueing analysis).

But…for now

Only look at the ***arrival process***

And how it evolves from a ***schedule***

# Start with a simple deterministic model of an appointment arrival process

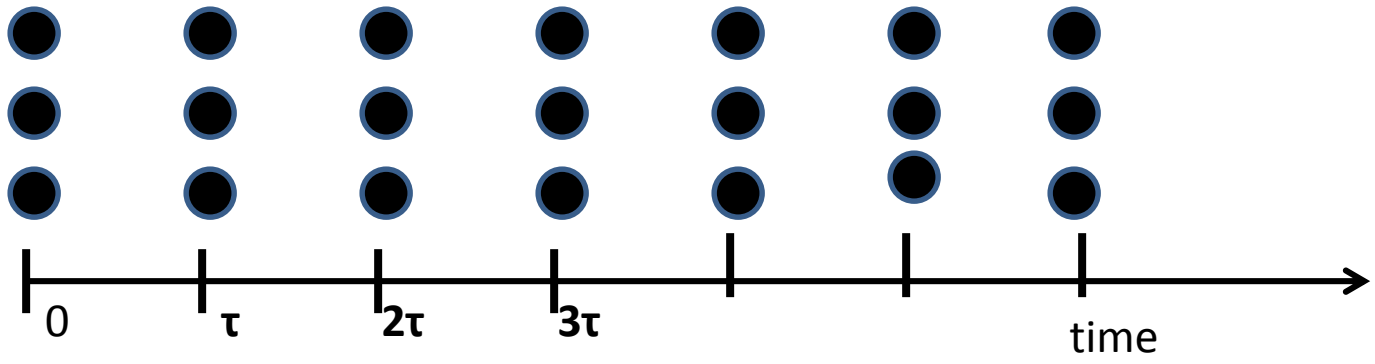
And investigate how realistic it is.

For our example, the evidence indicates that ***the data are neither consistent with the deterministic model nor an NHPP***.

We have developed new models.

# Deterministic Appointment Model

- Often Involving a **computerized appointment system**
- **Batches** of **β** customers arrive every **τ** minutes
  - Batch ***j*** arrives at time **(*j-1*) τ**
  - Total of **ν** batches
  - Daily Total Number of arrivals: $N = \nu \beta$
  - Total Time: $T = (\nu - 1)\,\tau$



# Random Schedule and Deviations From It

- Random Filling of Schedule over Time
- Random No Shows on the day
- Extra Unscheduled Arrivals
- Random Lateness or Earliness

So…

Is a deterministic model appropriate?

Or… Is an NHPP appropriate?

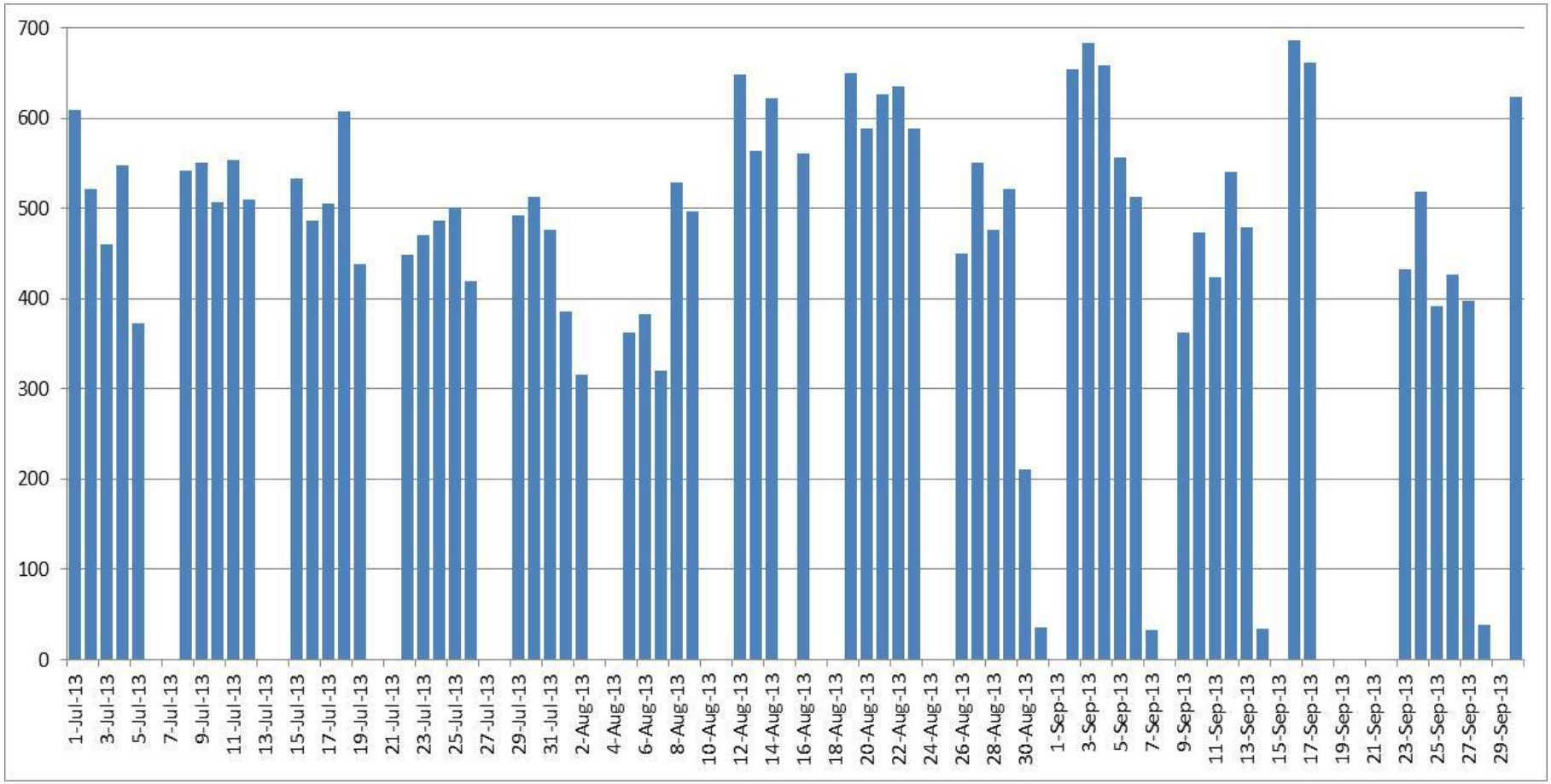
**No, we need something else.**

# DATA

## Endocrinology Clinic
## Samsung Medical Center, Seoul, Korea

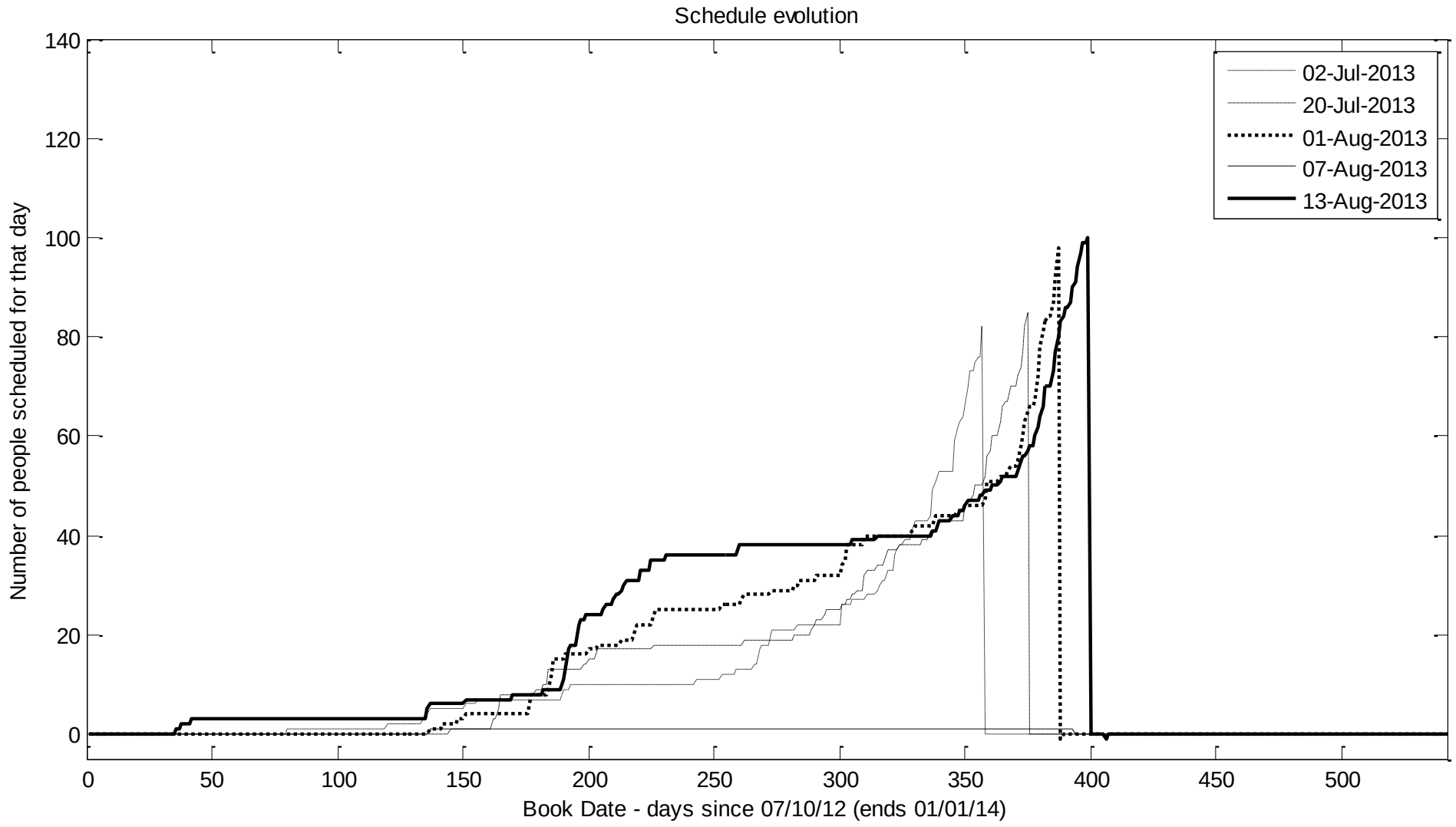
- Endocrinology: deals with the endocrine system (glands which excrete hormones into the blood stream)
- Three Months: 7/1/2013-9/30/2013
- Appointments to see one of 16 doctors
- Outpatient only (within one day)
- 39,253 entries
  - Scheduled day and time (by 10 minutes), when schedule made
  - Schedule evolves over time, including on the target day
  - Define “the schedule” as determined at the end of the previous day
  - **27800 show up**, 8500 cancel in advance, 3000 no shows

# Actual Daily Arrivals: 62 days
# 14 weeks, 7/1 to 9/30: 448 x 62 ≈ 27,800

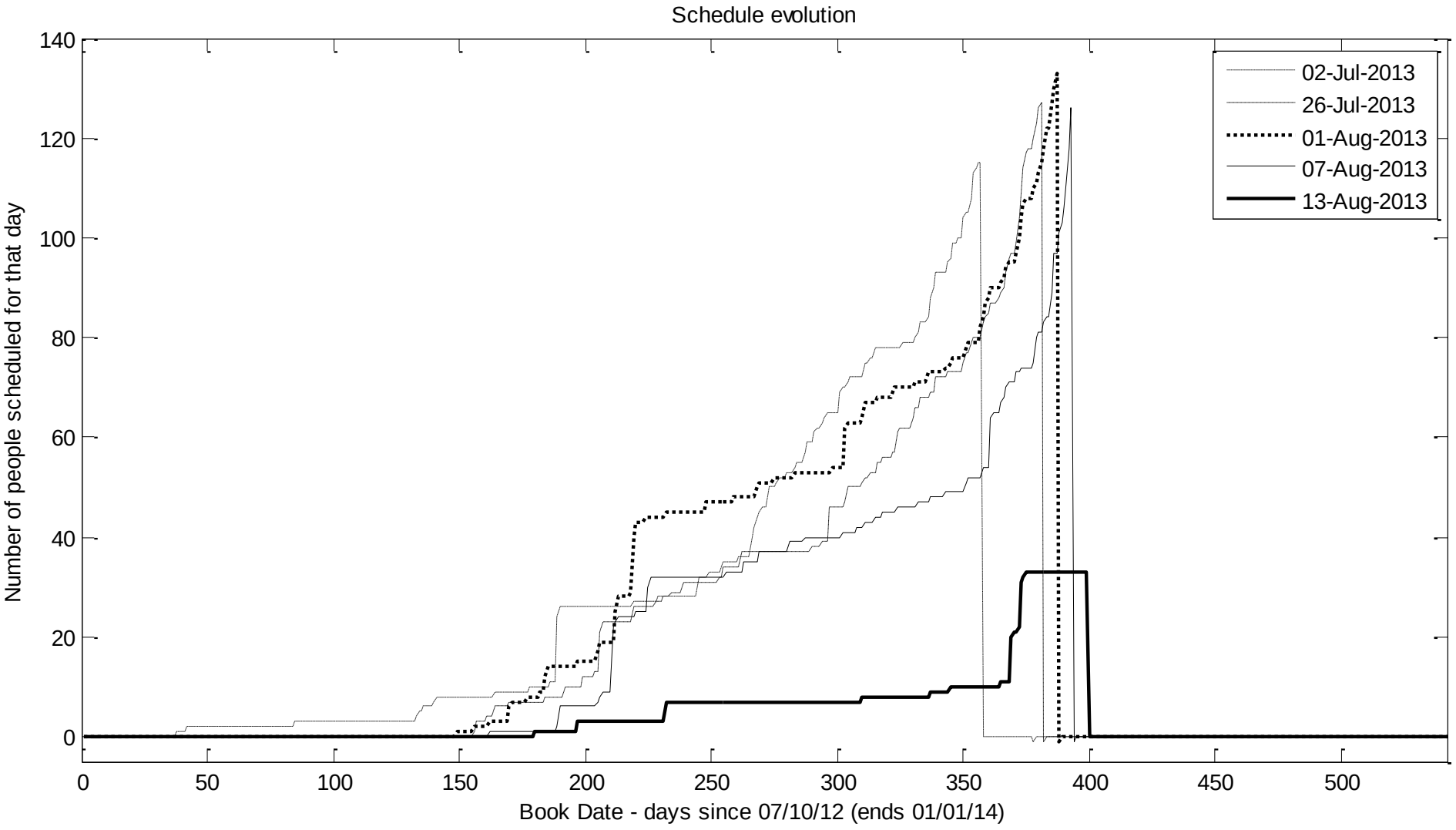



(Standard Deviation if Poisson is $(448)^{1/2} = 21$)

# Evolution of the Schedule for Dr. 1



# Evolution of the Schedule for Dr. 5



# The schedule evolves over time.

About 25% set the last two weeks

About 55% determined 2-13 weeks in advance (many at about 3 months)

About 20% determined more than 13 in advance.

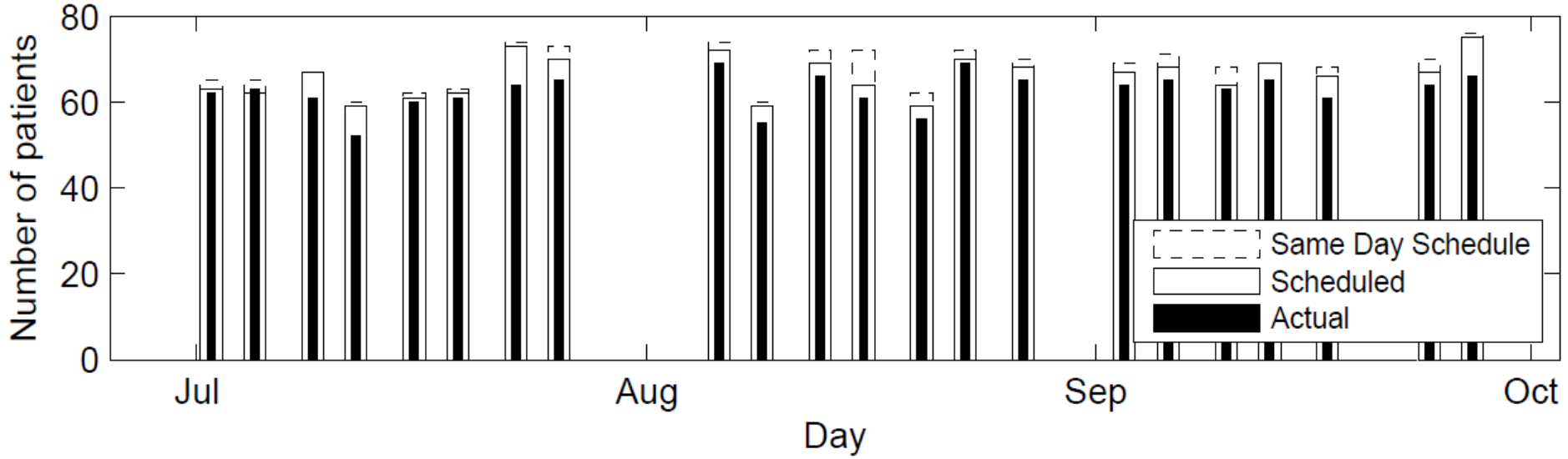
Focus on ONE DOCTOR:

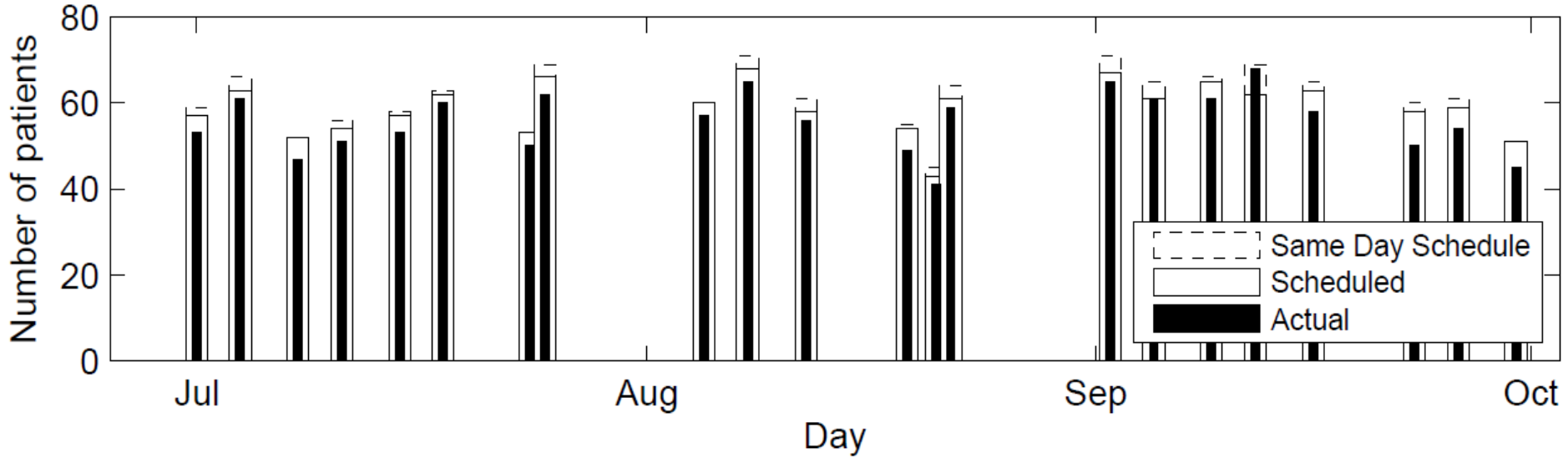
# Doctor 9

22 morning shifts

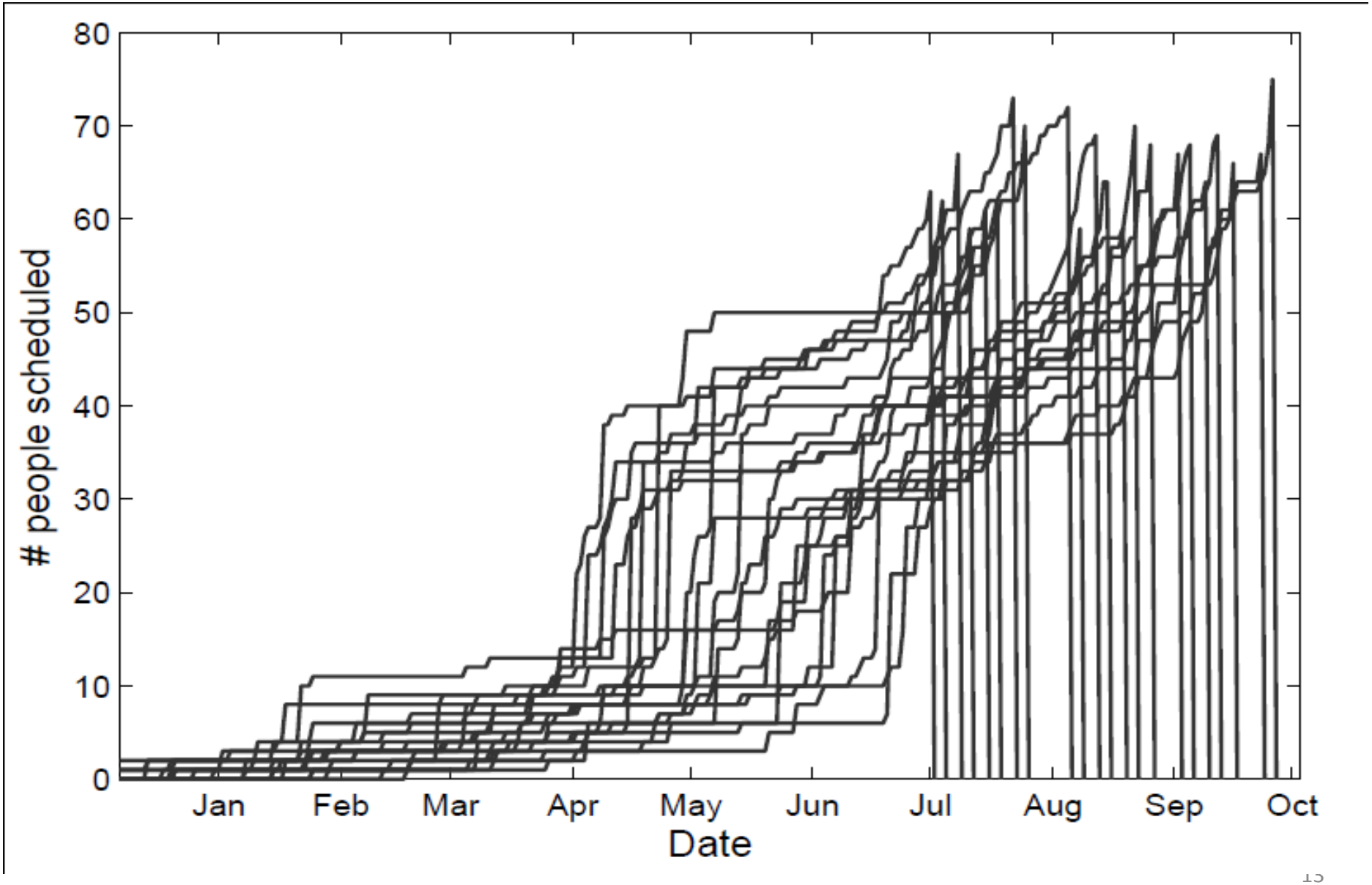
8:00am-1:00pm

# Daily totals for Dr. 9:  22 am shifts (top), 22 pm shifts (bottom)

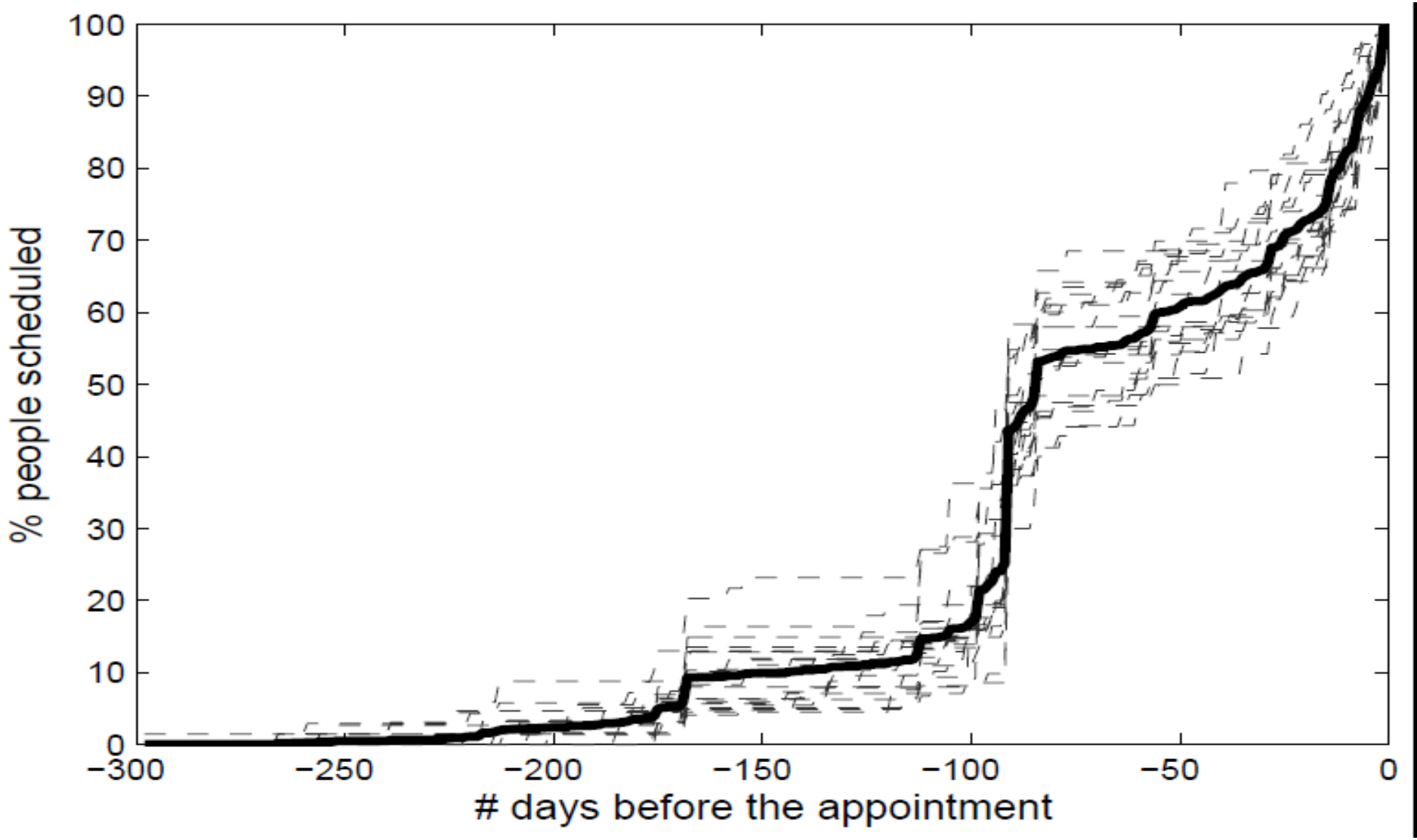


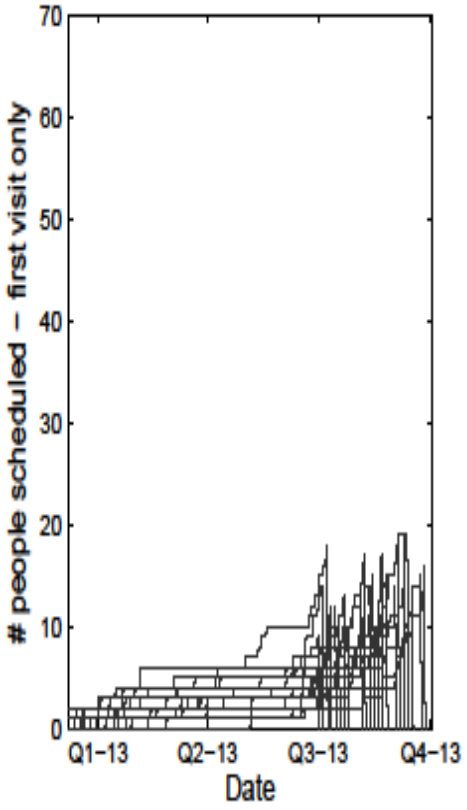
# Evolution of the Schedule for Dr. 9 Over a Year: 22 am shifts

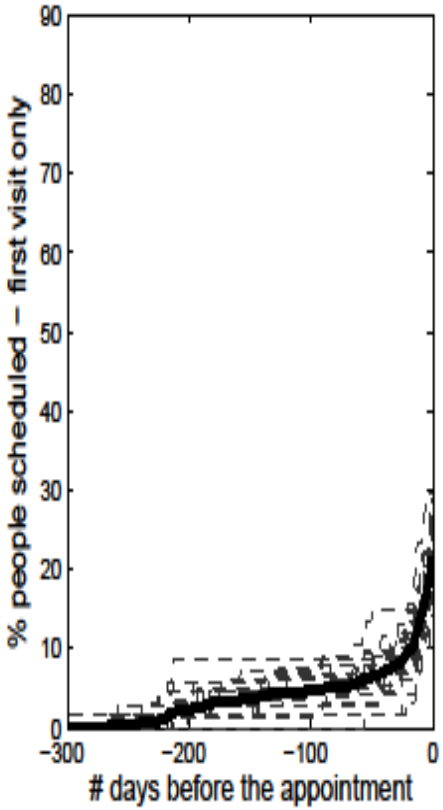


# When the Schedule Is Made: Number of Days Before

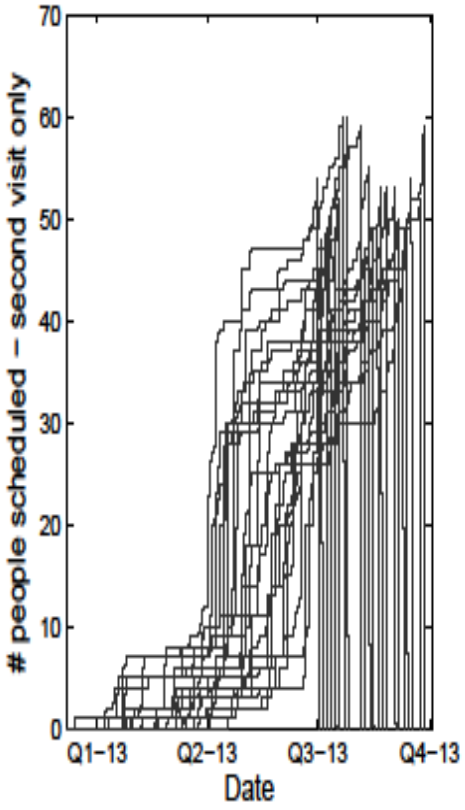


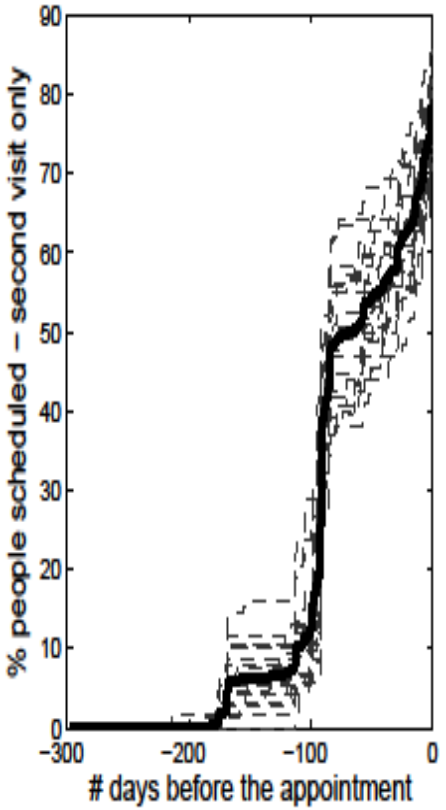
Evolution of Schedule for First Visits 22%




Evolution of Schedule for Repeat Visits 78%



# Let ***the schedule*** be ***defined*** by its state at the end of the previous day.

**The schedule**: Batches $B_{sj}$ scheduled to arrive in slot j at time (j-1)τ

No shows $B_{nj}$

Additional unscheduled arrivals $B_{uj}$
(scheduled on the same day)

Then $B_{aj} = B_{sj} - B_{nj} + B_{uj}$
(not counting deviations: earliness or lateness)

# Schedule for Doctor 9 for 22 Morning Shifts

| time slot | 22 days in July-October 2013 | | | | | | | | | | | | | | | | | | | | | | Avg | Var | Var/Avg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:00 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0.32 | 0.23 | 0.71 |
| 8:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 8:20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:50 | 3 | 4 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 1 | 3 | 2 | 1 | 4 | 2 | 4 | 4 | 2 | 4 | 5 | 4 | 3 | 3.41 | 1.30 | 0.38 |
| 9:00 | 3 | 4 | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 4 | 3 | 2 | 3 | 4 | 2 | 2.77 | 0.47 | 0.17 |
| 9:10 | 3 | 3 | 3 | 2 | 2 | 2 | 4 | 2 | 2 | 3 | 2 | 3 | 2 | 3 | 3 | 3 | 2 | 2 | 3 | 2 | 3 | 3 | 2.59 | 0.35 | 0.13 |
| 9:20 | 2 | 2 | 4 | 2 | 3 | 2 | 3 | 2 | 2 | 3 | 3 | 3 | 2 | 3 | 2 | 3 | 3 | 3 | 3 | 2 | 3 | 2 | 2.59 | 0.35 | 0.13 |
| 9:30 | 3 | 2 | 3 | 4 | 3 | 3 | 4 | 3 | 3 | 3 | 3 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2.77 | 0.47 | 0.17 |
| 9:40 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 3 | 2 | 3 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2.36 | 0.24 | 0.10 |
| 9:50 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 2.77 | 0.18 | 0.07 |
| 10:00 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 3 | 3 | 3 | 3 | 2.91 | 0.28 | 0.10 |
| 10:10 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.91 | 0.09 | 0.03 |
| 10:20 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 2 | 3 | 2 | 3 | 4 | 3 | 3 | 2.82 | 0.25 | 0.09 |
| 10:30 | 3 | 2 | 3 | 3 | 3 | 2 | 4 | 2 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 2 | 4 | 3 | 3 | 2.82 | 0.35 | 0.12 |
| 10:40 | 3 | 1 | 3 | 3 | 3 | 1 | 3 | 2 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 1 | 3 | 2 | 3 | 3 | 3 | 2 | 2.45 | 0.55 | 0.22 |
| 10:50 | 2 | 3 | 3 | 3 | 1 | 2 | 3 | 2 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 2.68 | 0.32 | 0.12 |
| 11:00 | 3 | 2 | 3 | 2 | 3 | 2 | 3 | 2 | 2 | 4 | 4 | 4 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 3 | 4 | 2.95 | 0.52 | 0.18 |
| 11:10 | 3 | 3 | 3 | 1 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 1 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 2.64 | 0.43 | 0.16 |
| 11:20 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 2.91 | 0.18 | 0.06 |
| 11:30 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 3 | 2 | 2.77 | 0.18 | 0.07 |
| 11:40 | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 1 | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 3 | 2.68 | 0.32 | 0.12 |
| 11:50 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 2 | 3 | 2 | 4 | 3 | 3 | 3 | 2 | 2 | 3 | 3 | 1 | 3 | 2.68 | 0.42 | 0.16 |
| 12:00 | 2 | 3 | 3 | 2 | 3 | 3 | 4 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 4 | 2.86 | 0.31 | 0.11 |
| 12:10 | 3 | 3 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 4 | 3 | 1 | 2 | 3 | 2 | 3 | 3 | 2.68 | 0.42 | 0.16 |
| 12:20 | 2 | 4 | 3 | 2 | 3 | 3 | 3 | 3 | 4 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 1 | 3 | 1 | 4 | 3 | 3 | 2.77 | 0.66 | 0.24 |
| 12:30 | 2 | 1 | 0 | 0 | 0 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 2 | 4 | 3 | 1 | 2 | 3 | 2.14 | 1.27 | 0.59 |
| 12:40 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 4 | 3 | 0 | 3 | 2 | 1 | 2 | 3 | 3 | 4 | 2 | 3 | 0 | 0 | 3 | 1.68 | 2.13 | 1.27 |
| 12:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 4 | 0 | 0 | 0 | 0 | 3 | 4 | 0 | 2 | 0 | 4 | 0 | 0 | 4 | 1.00 | 2.67 | 2.67 |
| 13:00 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| Daily Total | 63 | 62 | 67 | 59 | 61 | 62 | 73 | 70 | 72 | 59 | 69 | 64 | 59 | 70 | 68 | 67 | 68 | 64 | 69 | 66 | 67 | 75 | 66.09 | 21.32 | 0.32 |
| [8:50, 12:20] Total | 60 | 61 | 67 | 59 | 60 | 57 | 67 | 60 | 61 | 57 | 63 | 60 | 56 | 62 | 57 | 61 | 60 | 58 | 59 | 65 | 64 | 64 | 60.82 | 9.77 | 0.16 |
| All slot avg | 2.0 | 2.0 | 2.2 | 1.9 | 2.0 | 2.0 | 2.4 | 2.3 | 2.3 | 1.9 | 2.2 | 2.1 | 1.9 | 2.3 | 2.2 | 2.2 | 2.2 | 2.1 | 2.2 | 2.1 | 2.2 | 2.4 | 2.07 | 1.73 | 0.84 |
| All slot var | 1.5 | 1.9 | 2.2 | 1.9 | 1.8 | 1.5 | 1.8 | 1.3 | 1.5 | 1.7 | 1.5 | 1.5 | 1.6 | 1.5 | 1.3 | 1.7 | 1.8 | 1.6 | 1.6 | 2.2 | 1.8 | 1.6 | (across all days) | | |
| All slot var/avg | 0.7 | 1.0 | 1.0 | 1.0 | 0.9 | 0.8 | 0.8 | 0.6 | 0.6 | 0.9 | 0.7 | 0.7 | 0.8 | 0.6 | 0.6 | 0.8 | 0.8 | 0.8 | 0.7 | 1.1 | 0.8 | 0.7 | | | |
| [8:50, 12:20] avg | 2.7 | 2.8 | 3.0 | 2.7 | 2.7 | 2.6 | 3.0 | 2.7 | 2.8 | 2.6 | 2.9 | 2.7 | 2.5 | 2.8 | 2.6 | 2.8 | 2.7 | 2.6 | 2.7 | 3.0 | 2.9 | 2.9 | 2.76 | 0.42 | 0.15 |
| [8:50, 12:20] var | 0.2 | 0.6 | 0.3 | 0.5 | 0.4 | 0.4 | 0.4 | 0.3 | 0.4 | 0.5 | 0.2 | 0.3 | 0.5 | 0.3 | 0.4 | 0.4 | 0.7 | 0.3 | 0.4 | 0.7 | 0.4 | 0.4 | (across all days) | | |
| [8:50, 12:20] var/avg | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.3 | 0.1 | 0.2 | 0.2 | 0.1 | 0.1 | | | |

# A High-Demand Service System with Overloaded and At-Capacity Days

**Distribution of Batch Size $B_s$ in the Main interval [8:50,12:20]**

| | $\hat{P}(B_s = k)$ | | | | |
|---|---|---|---|---|---|
| number $k$ | 1 | 2 | 3 | 4 | 5 |
| 10 at-capacity days | 0.04 | 0.25 | 0.63 | 0.07 | 0.01 |
| 12 overloaded days | 0.02 | 0.27 | 0.63 | 0.08 | |
| All days | 0.03 | 0.26 | 0.63 | 0.08 | 0.004 |

**Distribution of Number After the Main interval, $N_o$**

| $\hat{P}(N_o = k)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0.30 | 0.20 | 0.30 | 0.10 | 0.10 | | | | | | |
| | | | | | 0.25 | 0.17 | | 0.25 | | 0.33 |
| 0.14 | 0.09 | 0.14 | 0.05 | 0.05 | 0.14 | 0.09 | | 0.14 | | 0.18 |

# No-Shows for Doctor 9 for 22 Morning Shifts

| time slot | 22 days in July-October 2013 | | | | | | | | | | | | | | | | | | | | | | Avg | Var | Var/Avg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:00 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0.32 | 0.23 | 0.71 |
| 8:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 8:20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:50 | 1 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0.23 | 0.28 | 1.23 |
| 9:00 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.18 | 0.16 | 0.86 |
| 9:10 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.27 | 0.30 | 1.11 |
| 9:20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.18 | 0.25 | 1.38 |
| 9:30 | 0 | 0 | 0 | 2 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.18 | 0.25 | 1.38 |
| 9:40 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0.18 | 0.25 | 1.38 |
| 9:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 10:00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0.27 | 0.30 | 1.11 |
| 10:10 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 | 0 | 0.32 | 0.32 | 1.01 |
| 10:20 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0.23 | 0.18 | 0.81 |
| 10:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.18 | 0.16 | 0.86 |
| 10:40 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0.23 | 0.18 | 0.81 |
| 10:50 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0.27 | 0.30 | 1.11 |
| 11:00 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0.32 | 0.23 | 0.71 |
| 11:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0.18 | 0.16 | 0.86 |
| 11:20 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0.23 | 0.18 | 0.81 |
| 11:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0.23 | 0.18 | 0.81 |
| 11:40 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.23 | 0.18 | 0.81 |
| 11:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 12:00 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 2 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | 0.55 | 0.45 | 0.83 |
| 12:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 12:20 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 12:30 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 12:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0.09 | 0.09 | 0.95 |
| 12:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0.05 | 0.05 | 1.00 |
| 13:00 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| Daily Total | 3 | 2 | 6 | 8 | 2 | 2 | 10 | 7 | 5 | 4 | 6 | 10 | 6 | 2 | 5 | 5 | 6 | 5 | 4 | 7 | 5 | 10 | 5.45 | 6.35 | 1.17 |
| [8:50, 12:20] Total | 2 | 2 | 6 | 8 | 1 | 2 | 8 | 4 | 4 | 4 | 4 | 10 | 6 | 1 | 4 | 5 | 6 | 5 | 4 | 7 | 4 | 7 | 4.73 | 5.64 | 1.19 |
| All slot avg | 2.0 | 2.0 | 2.2 | 1.9 | 2.0 | 2.0 | 2.4 | 2.3 | 2.3 | 1.9 | 2.2 | 2.1 | 1.9 | 2.3 | 2.2 | 2.2 | 2.2 | 2.1 | 2.2 | 2.1 | 2.2 | 2.4 | 0.17 | 0.17 | 1.00 |
| All slot var | 1.5 | 1.9 | 2.2 | 1.9 | 1.8 | 1.5 | 1.8 | 1.3 | 1.5 | 1.7 | 1.5 | 1.5 | 1.6 | 1.5 | 1.3 | 1.7 | 1.8 | 1.6 | 1.6 | 2.2 | 1.8 | 1.6 | (across all days) | | |
| All slot var/avg | 0.7 | 1.0 | 1.0 | 1.0 | 0.9 | 0.8 | 0.8 | 0.6 | 0.6 | 0.9 | 0.7 | 0.7 | 0.8 | 0.6 | 0.6 | 0.8 | 0.8 | 0.8 | 0.7 | 1.1 | 0.8 | 0.7 | | | |
| [8:50, 12:20] avg | 2.7 | 2.8 | 3.0 | 2.7 | 2.7 | 2.6 | 3.0 | 2.7 | 2.8 | 2.6 | 2.9 | 2.7 | 2.5 | 2.8 | 2.6 | 2.8 | 2.7 | 2.6 | 2.7 | 3.0 | 2.9 | 2.9 | 0.21 | 0.21 | 0.98 |
| [8:50, 12:20] var | 0.2 | 0.6 | 0.3 | 0.5 | 0.4 | 0.4 | 0.4 | 0.3 | 0.4 | 0.5 | 0.2 | 0.3 | 0.5 | 0.3 | 0.4 | 0.4 | 0.7 | 0.3 | 0.4 | 0.7 | 0.4 | 0.4 | (across all days) | | |
| [8:50, 12:20] var/avg | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.3 | 0.1 | 0.2 | 0.2 | 0.1 | 0.1 | | | |

# Unscheduled Arrivals for Doctor 9 on the 22 Morning Shifts

| time slot | 22 days in July-October 2013 | | | | | | | | | | | | | | | | | | | | | | Avg | Var | Var/Avg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.05 | 0.05 | 1.00 |
| 8:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 8:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 9:00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0.09 | 0.09 | 0.95 |
| 9:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 9:20 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 9:30 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 9:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 9:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 10:00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 10:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 10:20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0.14 | 0.12 | 0.90 |
| 10:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 10:40 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 10:50 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 11:00 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 11:10 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 11:20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 11:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 11:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 11:50 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.09 | 0.09 | 0.95 |
| 12:00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | | |
| 12:10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 12:20 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 1.00 |
| 12:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.14 | 0.12 | 0.90 |
| 12:40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.14 | 0.22 | 1.60 |
| 12:50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| 13:00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.09 | 0.95 |
| Daily Total | 2 | 3 | 0 | 1 | 1 | 1 | 1 | 3 | 2 | 1 | 3 | 8 | 3 | 2 | 2 | 2 | 3 | 4 | 0 | 2 | 3 | 1 | 2.18 | 2.82 | 1.29 |
| [8:50, 12:20] Total | 2 | 3 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 2 | 1 | 2 | 2 | 3 | 3 | 0 | 1 | 2 | 1 | 1.68 | 1.27 | 0.76 |

# Daily Totals

$N_s$ = number scheduled, $N_a$ = number arrived
$N_n$ = number no-shows, $N_u$ = number unscheduled

**Full am shift**: $N_a = N_s - N_n + N_u$

E[$N_s$] = 66.1, E[$N_n$] = 5.5, E[$N_u$] = 2.0, E[$N_a$] = 62.6

Var($N_s$) = 21.9 > Var($N_a$) = 17.4

Dispersion: D = Var/Mean

$D_s$ = 21.9/66.1 = 0.33, $D_a$ = 17.4/62.6 = 0.278

**Main time interval** [8:50, 12:20]:

E[$N_s$] = 60.8 while Var($N_s$) = 9.8

# View of Daily Totals

**With appointments**: Not constant, but predictable variability, so **Under-Dispersion**
(less variable than Poisson: ***D=V/M = 0.3 < 1***)

**With Call Centers**: often strong day-to-day variation, so **Over-Dispersion**
(more variable than Poisson: ***D=V/M >1***)

# Random Earliness or Lateness

Let $\boldsymbol{X}$ be the amount of time late.

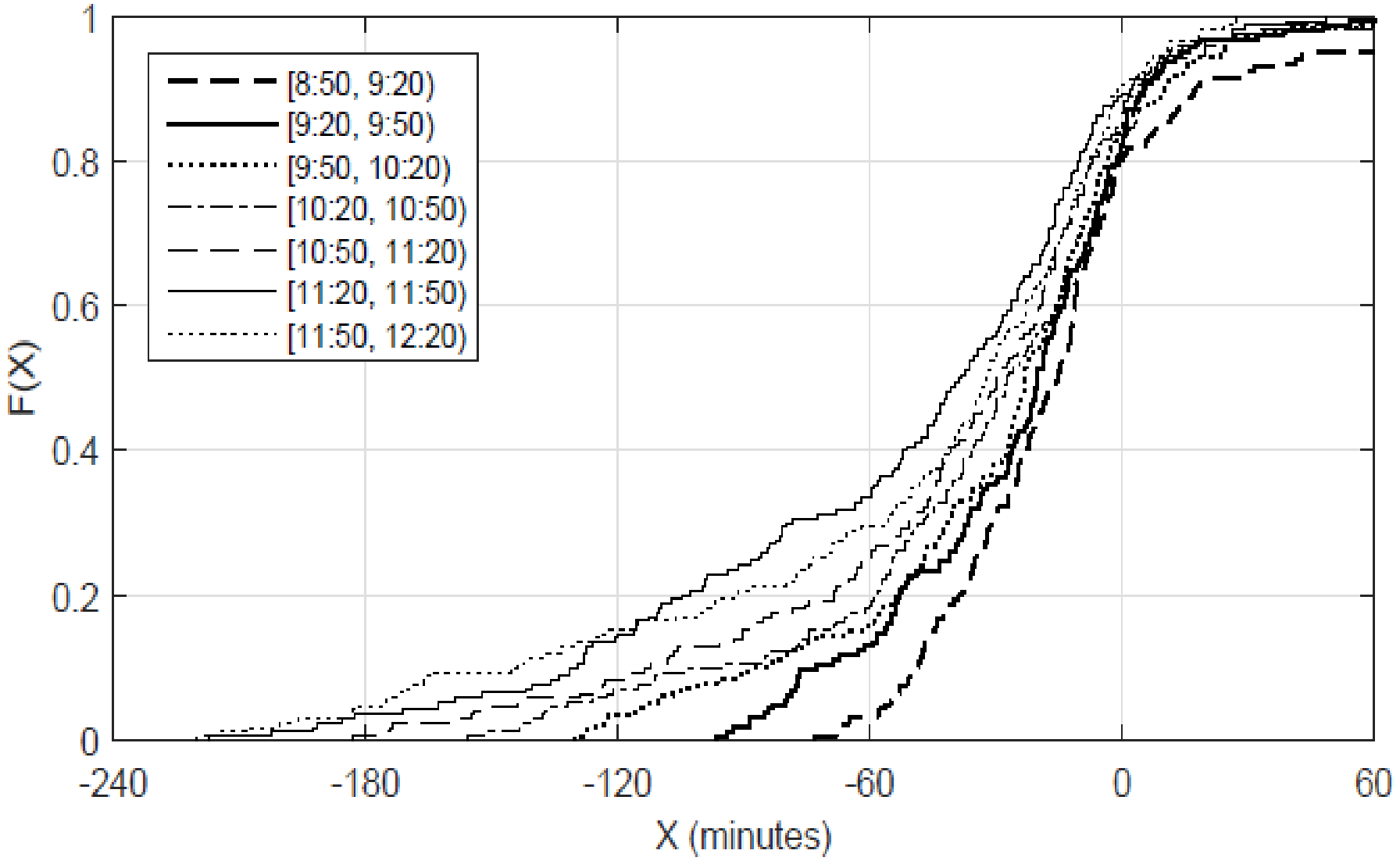
Let $\boldsymbol{F}(t) = P(\boldsymbol{X} \leq t)$ be the cdf.

(X < 0 means arrive early)

# Average numbers for Dr. 9 in each 30-minute subinterval within the main time interval [8:50,12:20]

| Interval | Avg # Scheduled | % No-show | % Late | % (Late>15 min) |
|---|---|---|---|---|
| [8:50, 9:20) | 8.8±0.7 | 7.9±4.8 | 21.2±6.9 | 12.3±5.5 |
| [9:20, 9:50) | 7.7±0.5 | 6.9±4.6 | 16.7±6.1 | 4.8±3.4 |
| [9:50, 10:20) | 8.6±0.4 | 6.8±4.4 | 15.0±6.7 | 6.4±3.4 |
| [10:20, 10:50) | 8.1±0.6 | 7.9±3.2 | 17.6±5.0 | 3.3±2.9 |
| [10:50, 11:20) | 8.3±0.5 | 9.0±3.9 | 13.6±4.4 | 5.4±3.9 |
| [11:20, 11:50) | 8.4±0.3 | 7.9±3.7 | 10.4±4.7 | 3.9±3.0 |
| [11:50, 12:20) | 8.2±0.5 | 9.3±4.1 | 9.5±5.4 | 3.8±3.5 |
| [8:50, 12:20) | 58.0±1.3 | 8.0±1.7 | 15.0±1.5 | 5.8±1.6 |

# Dr. 9: Lateness empirical cdf's in each 30-minute interval



# Within the Day
# Conditional on the daily total

If there is significant variability due to no shows and lateness,
Then the arrival process may be NHPP within the day,
*conditional on the daily total.*
**[Key math: generalization of conditional uniform property]**

Appointments might be like call centers in that way.

Even though daily totals are very different.

# Appointment arrival data passed the KS Tests of NHPP when viewed properly.

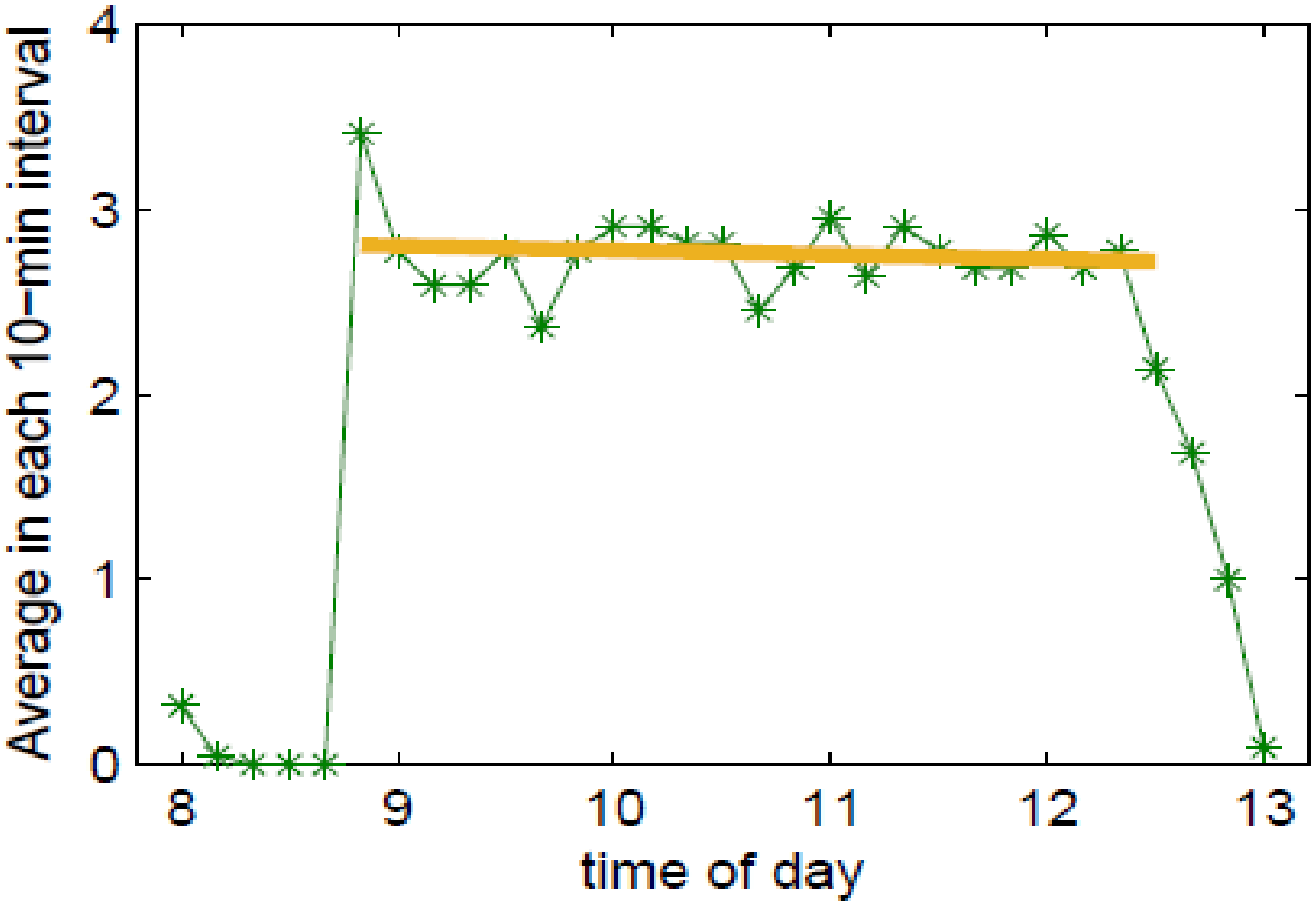
Next Move to the arrival rate function

# Overall Arrival Process Model

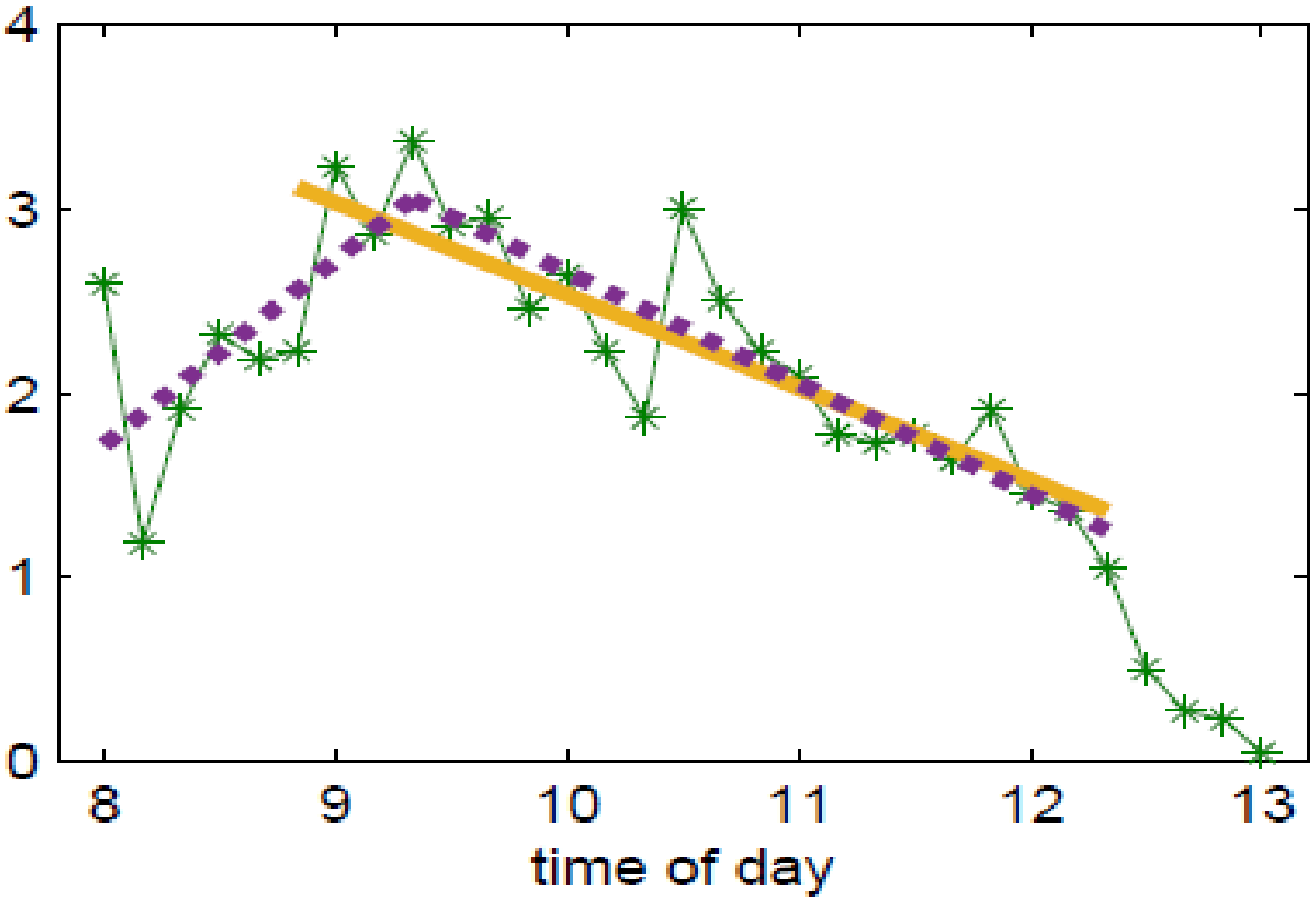
**Daily Totals** : Binomial or Gaussian with given mean and variance

**Within the day, conditional on the daily total**:
NHPP with deterministic rate

**Average number of patients *scheduled* for Dr. 9 in each ten-minute interval of the 22 am shifts**



**Average number of patient *arrivals* for Dr. 9**
**in each ten-minute interval of the 22 am shifts**



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